Kittel

2.4 (a) $|F|^2 = F^* F = \dfrac{1-\exp[iM(\vec{a}\cdot\Delta\vec{k})]}{1-\exp[i(\vec{a}\cdot\Delta\vec{k})]} \dfrac{1-\exp[-iM(\vec{a}\cdot\Delta\vec{k})]}{1-\exp[-i(\vec{a}\cdot\Delta\vec{k})]}$

let x denote $\vec{a}\cdot\Delta\vec{k}$.

$$F^* F = \frac{1+1-\exp[iMx]-\exp[-iMx]}{1+1-\exp[ix]-\exp[-ix]}.$$

$$= \frac{2-2\cos(Mx)}{2-2\cos(x)}.$$

$$= \frac{2-2\left[\cos^2\left(\frac{Mx}{2}\right)-\sin^2\left(\frac{Mx}{2}\right)\right]}{2-2\left[\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)\right]}.$$

$$= \frac{2-2\cos^2\left(\frac{Mx}{2}\right)+2\sin^2\left(\frac{Mx}{2}\right)}{2-2\cos^2\left(\frac{x}{2}\right)+2\sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{4\sin^2\left(\frac{Mx}{2}\right)}{4\sin^2\left(\frac{x}{2}\right)} = \boxed{\frac{\sin^2\left[\frac{1}{2}M(\vec{a}\cdot\Delta\vec{k})\right]}{\sin^2\left[\frac{1}{2}(\vec{a}\cdot\Delta\vec{k})\right]}}.$$

(b) Since $\overbrace{\text{zeros of } \sin(x)}$ has $\overbrace{\text{periodicity of } \pi}$, it's obvious that the next zero of $\sin\left[\frac{1}{2}M(\vec{a}\cdot\Delta\vec{k})\right]$ after $2\pi h$ would be $\frac{2\pi}{2}$.

thus $\varepsilon = 2\pi/M$.

ε has units of $\frac{\text{phase}}{\text{distance}}$, this checks out that the width of diffraction max is $\frac{1}{M}$.